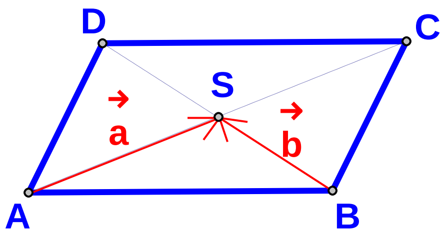
Nastavna cjelina: **VEKTORI** **Razred: III**

ISPITNI ZADACI

1. Dan je paralelogram ABCD. Točka S je sjecište dijagonala. Izrazi vektore $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}, 2\overrightarrow{AC} + \frac{1}{2}\overrightarrow{BC}$ kao linearnu kombinaciju vektora $\vec{a} = \overrightarrow{AS}$ i $\vec{b} = \overrightarrow{BS}$. *(~5 bodova)*



$\vec{a} = \overrightarrow{AS}$

$\vec{b} = \overrightarrow{BS}$

$$\overrightarrow{AB} = \overrightarrow{AS} - \overrightarrow{BS} = \vec{a} - \vec{b} \qquad (1)$$

$$\overrightarrow{BC} = \overrightarrow{BS} + \overrightarrow{SC} = \overrightarrow{BS} + \overrightarrow{AS} = \vec{b} + \vec{a} \qquad (1)$$

$$\overrightarrow{AC} = \frac{1}{2}\overrightarrow{AS} = \frac{1}{2}\vec{a} \qquad (1)$$

$$2\overrightarrow{AC} + \frac{1}{2}\overrightarrow{BC} = 2\left(\vec{a} - \vec{b}\right) + \frac{1}{2}\left(\vec{b} + \vec{a}\right) = 2\vec{a} - 2\vec{b} + \frac{1}{2}\vec{b} + \frac{1}{2}\vec{a}$$

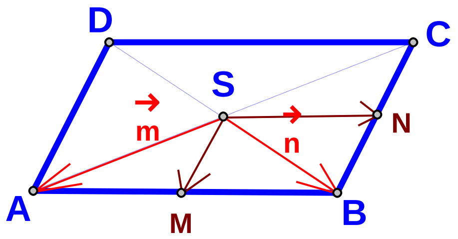
$$= \vec{a}\left(2 + \frac{1}{2}\right) + \vec{b}\left(-2 + \frac{1}{2}\right) = \vec{a}\,\frac{4+1}{2} + \vec{b}\,\frac{-4+1}{2}$$

$$= \frac{5}{2}\vec{a} - \frac{3}{2}\vec{b}$$

(2)

ISPITNI ZADACI

2. Zadan je paralelogram ABCD, točka S je sjecište njegovih dijagonala, a točke M i N su polovište stranica $\overline{AB}$ i $\overline{BC}$. Pomoću vektora
$\vec{m} = \overrightarrow{SA}$ i $\vec{n} = \overrightarrow{SB}$ prikaži vektore $\overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{SM}$ i $\overrightarrow{SN}$. *(~5 bodova)*



$\vec{m} = \overrightarrow{SA}$

$\vec{n} = \overrightarrow{SB}$

$\overrightarrow{BC} = -(\vec{m} + \vec{n})$

$\overrightarrow{DC} = \vec{n} - \vec{m}$

$\overrightarrow{SM} = \vec{n} + \left(-\frac{1}{2}\overrightarrow{DC}\right) = \frac{1}{2}\vec{n} + \frac{1}{2}\vec{m}$

$\overrightarrow{SN} = -\vec{m} - \frac{1}{2}\overrightarrow{BC} = -\frac{1}{2}\vec{m} + \frac{1}{2}\vec{n}$

3 Prikaži vektor $\vec{c} = \vec{i} - 6\vec{j}$ kao linearnu kombinaciju vektora
$\vec{a} = \vec{i} - 2\vec{j}$ i $\vec{b} = 2\vec{i} - 4\vec{j}$. ne može – krivo prepisan zadatak

4. Vektor $\vec{c} = 9\vec{i} + 4\vec{j}$ prikaži kao linearnu kombinaciju vektora
$\vec{a} = 2\vec{i} - 3\vec{j}$ i $\vec{b} = \vec{i} + 2\vec{j}$ *(~2 boda)*

$\vec{c} = 2\vec{a} + 5\vec{b} = 2(2\vec{i} - 3\vec{j}) + 5(\vec{i} + 2\vec{j}) = 4\vec{i} - 6\vec{j} + 5\vec{i} + 10\vec{j}$

$9\vec{i} + 4\vec{j} = 9\vec{i} + 4\vec{j}$

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5. Vektor $\vec{c} = \vec{\imath} + 5\,\vec{j}$ prikaži kao linearnu kombinaciju vektora $\vec{a} = \vec{\imath} - \vec{j}$ i $\vec{b} = 2\vec{\imath} + \vec{j}$.

$\vec{c} = \vec{\imath} + 5\,\vec{j}$

$\vec{a} = \vec{\imath} - \vec{j}$

$\vec{b} = 2\vec{\imath} + \vec{j}$

$X \cdot (\vec{\imath} - \vec{j}) + Y \cdot (2\,\vec{\imath} + \vec{j}) = \vec{c}$

Kliko iznosi X i Y da dobijemo vektor $\vec{c}$?

X = -3
Y = 2

$-3 \cdot (\vec{\imath} - \vec{j}) + 2 \cdot (2\,\vec{\imath} + \vec{j}) = \vec{\imath} + 5\,\vec{j}$

$-3\vec{\imath} + 3\,\vec{j} + 4\,\vec{\imath} + 2\,\vec{j} = \vec{\imath} + 5\,\vec{j}$

$(-3 + 4)\,\vec{\imath} + (3 + 2)\,\vec{j} = \vec{\imath} + 5\,\vec{j}$

$(-3 + 4)\,\vec{\imath} + (3 + 2)\,\vec{j} = \vec{\imath} + 5\,\vec{j}$

$1\,\vec{\imath} + 5\,\vec{j} = \vec{\imath} + 5\,\vec{j}$

$\vec{\imath} + 5\,\vec{j} = \vec{\imath} + 5\,\vec{j}$

6. Dan je pravilni šesterokut ABCDEF. Točka S je njegovo središte. Izrazi vektore $\overrightarrow{BC}, \overrightarrow{DE}, \overrightarrow{EF}$ i $\overrightarrow{BE}$ $\vec{a} = \overrightarrow{AB}$ i $\vec{b} = \overrightarrow{AC}$. *(~5 bodova)*

$\overrightarrow{BC} = -\vec{a} + \vec{b}$

$\overrightarrow{DE} = -\overrightarrow{AB} = -\vec{a}$

$\overrightarrow{EF} = -\vec{a}$

$\overrightarrow{SN} = 2\,\vec{a}$

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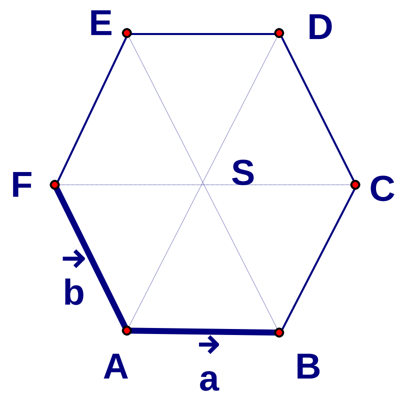
7. Dan je pravilni šesterokut ABCDEF. Ako je $\vec{a} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{AF}$ izrazi pomoću $\vec{a}$ i $\vec{b}$ vektora $\overrightarrow{CE}$, $\overrightarrow{EF}$, $\overrightarrow{BD}$ i $\overrightarrow{BE}$ *(~5 bodova)*



$\overrightarrow{CE} = \overrightarrow{BF} = \vec{b} - \vec{a}$

$\overrightarrow{EF} = -\vec{a} - \vec{b}$

$\overrightarrow{BD} = \vec{a} + 2\vec{b}$

$\overrightarrow{BE} = 2\,\vec{b}$

8. Odredi $\propto$ tako da vektori $\vec{a} = 4\,\vec{\imath} - \vec{\jmath}$ i $\vec{b} = \propto \vec{\imath} + 2\,\vec{\jmath}$ budu kolinearni. *(~2 boda)*

$\vec{a} = k\;\vec{b}$

$$4\,\vec{\imath} - \vec{\jmath} = k\,(\propto \vec{\imath} + 2\,\vec{\jmath})$$

$$4\,\vec{\imath} - \vec{\jmath} = k\,\propto \vec{\imath} + 2\,k\,\vec{\jmath}$$

$$4\,\vec{\imath} = k\,\propto \vec{\imath}$$

$$4 = k\,\propto$$

$$\propto = \frac{4}{k}$$

$-1 = 2\,k$

$-1 = 2\,k$

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$$k = -\frac{1}{2}$$

9. Odredi vektor $\vec{b}$ kolinearan s $\vec{a}$, ako je $\vec{a} = -2\,\vec{\imath} + \vec{\jmath}$, $|\vec{b}| = 3\sqrt{5}$. Početak vektora $\vec{a}$ i vektora $\vec{b}$ su u ishodištu koordinatnog sustava.

$$\vec{a} = -2\,\vec{\imath} + \vec{\jmath}$$

$$|\vec{b}| = 3\sqrt{5}$$

$$|\vec{b}| = \sqrt{{b_x}^2 + {b_y}^2} = 3\sqrt{5}$$

Kolinearni vektori imaju isti smjer.

$$(3\sqrt{5})^2 = {b_x}^2 + {b_y}^2$$

$$9 \cdot 5 = {b_x}^2 + {b_y}^2$$

$$45 = {b_x}^2 + {b_y}^2$$

$${b_x}^2 = 45 - {b_y}^2 \qquad (1)$$

$$\vec{a} = k\,\vec{b}$$

$$-2\,\vec{\imath} + \vec{\jmath} = k\,(b_x\,\vec{\imath} + b_y\,\vec{\jmath})$$

$$-2\,\vec{\imath} + \vec{\jmath} = k\,b_x\vec{\imath} + k\,b_y\,\vec{\jmath}$$

$$-2\,\vec{\imath} = k\,b_x\vec{\imath}$$

$$-2 = k\,b_x /^2$$

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$$(-2)^2 = k^2 {b_x}^2$$
$$4 = k^2 {b_x}^2$$
$$4 = k^2 \left(45 - {b_y}^2\right)$$

$$\vec{j} = k\, b_y\, \vec{j}$$
$$1 = k\, b_y$$
$$k = \frac{1}{b_y} \quad /^2$$
$$k^2 = \frac{1}{{b_y}^2} \qquad (2)$$

$$4 = k^2 {b_x}^2$$
$$4 = \frac{1}{{b_y}^2}\left(45 - {b_y}^2\right)$$
$$4 = \frac{45}{{b_y}^2} - \frac{{b_y}^2}{{b_y}^2}$$

$$4 = \frac{45}{{b_y}^2} - \frac{{b_y}^2}{{b_y}^2} = \frac{45}{{b_y}^2} - 1$$

$$\frac{45}{{b_y}^2} = 4 + 1$$

$$\frac{45}{{b_y}^2} = 5$$

$${b_y}^2 = \frac{45}{5}$$

$${b_y}^2 = 9 \;/ \pm\sqrt{}$$

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$$b_{y_1} = 3$$
$$b_{y_2} = -3$$

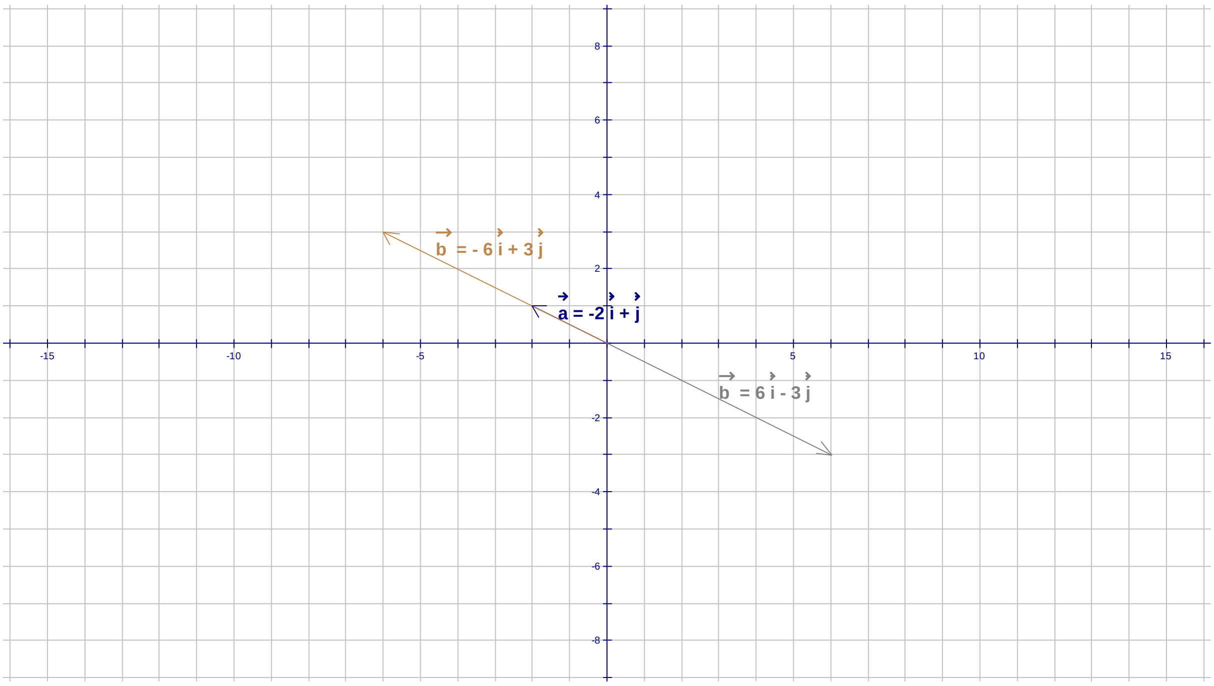
$$b_x^{\ 2} = 45 - b_y^{\ 2}$$
$$b_{x_{1,2}}^{\ \ 2} = 45 - b_y^{\ 2} = 45 - 3^2 = 45 - 9 = 36 \ / \mp\sqrt{\ }$$
$$b_{x_{1,2}} = \mp\sqrt{36} = \mp 6$$

$$b_{x_1} = -6$$
$$b_{x_2} = 6$$

$$\vec{b} = -6\,\vec{\imath} + 3\,\vec{\jmath}$$

ili

$$\vec{b} = 6\,\vec{\imath} - 3\,\vec{\jmath}$$



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10. Nađi kut između vektora $\vec{a} = \vec{i} + 3\,\vec{j}$ i $\vec{b} = 4\vec{i} - 2\,\vec{j}$.
(~4 boda)

$\vec{a} = a_x\,\vec{i} + a_y\,\vec{j}$
$\vec{b} = b_x\vec{i} + b_y\,\vec{j}$

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{{a_x}^2 + {a_y}^2} \cdot \sqrt{{b_x}^2 + {b_y}^2}}$$

$$\cos\varphi = \frac{1 \cdot 4 + 3 \cdot (-2)}{\sqrt{1^2 + 3^2} \cdot \sqrt{4^2 + (-2)^2}} = \frac{4 - 6}{\sqrt{1+9} \cdot \sqrt{16+4}} = \frac{-2}{\sqrt{10} \cdot \sqrt{20}}$$

$$\cos\varphi = = \frac{-2}{\sqrt{200}} = \frac{-2}{10\sqrt{2}} = \frac{-1}{5\sqrt{2}} = -\frac{\sqrt{2}}{10}$$

$\cos\varphi = -0.141421356$

$\varphi = 98°\,7'\,48.37''$

11. Koliki kut zatvaraju vektori $\vec{a} + \vec{b}$ i $\vec{a} - \vec{b}$, ako je $\vec{a} = \vec{i} + 4\,\vec{j}$ i $\vec{b} = -3\,\vec{i} + 2\,\vec{j}$.

$\vec{a} = \vec{i} + 4\,\vec{j}$
$\vec{b} = -3\,\vec{i} + 2\,\vec{j}$

$$\vec{a} + \vec{b} = (\vec{i} + 4\,\vec{j}) + (-3\,\vec{i} + 2\,\vec{j}) = \vec{i} + 4\vec{j} + 3\vec{i} - 2\vec{j}$$
$$= \big(1 + (-3)\big)\vec{i} + (4 + 2)\vec{j} = (1 - 3)\vec{i} + 6\,\vec{j} = -2\vec{i} + 6\,\vec{j}$$

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$$\vec{a}-\vec{b} = (\vec{\imath}+4\vec{\jmath}) - (-3\vec{\imath}+2\vec{\jmath}) = \vec{\imath}+4\vec{\jmath}+3\vec{\imath}-2\vec{\jmath}$$
$$= (1+3)\vec{\imath} + (4-2)\vec{\jmath} = 4\vec{\imath}+2\vec{\jmath}$$

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{{a_x}^2+{a_y}^2}\cdot\sqrt{{b_x}^2+{b_y}^2}}$$

$\vec{a}+\vec{b} = -2\vec{\imath}+6\vec{\jmath}$

$\vec{a}-\vec{b} = 4\vec{\imath}+2\vec{\jmath}$

$a_x = -2$
$a_y = 6$
$b_x = 4$
$b_y = 2$

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{{a_x}^2+{a_y}^2}\cdot\sqrt{{b_x}^2+{b_y}^2}}$$

$$\cos\varphi = \frac{-2\cdot 4+6\cdot 2}{\sqrt{(-2)^2+6^2}\cdot\sqrt{4^2+2^2}} = \frac{-8+12}{\sqrt{4+36}\cdot\sqrt{16+4}} = \frac{4}{\sqrt{40}\cdot\sqrt{20}}$$

$$\cos\varphi = \frac{4}{2\sqrt{10}\cdot 2\sqrt{5}} = \frac{1}{\sqrt{10}\cdot\sqrt{5}} = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

$\cos\varphi = 0.141421356$

$\varphi = 81^\circ\, 52'\, 11.63''$

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12. Koliki kut zatvaraju vektori $\vec{a}+\vec{b}$ i $\vec{a}-\vec{b}$, ako je $\vec{a}=-2\vec{i}+3\vec{j}$ i $\vec{b}=\vec{i}-\vec{j}$.

$$\vec{a}=-2\vec{i}+3\vec{j}$$
$$\vec{b}=\vec{i}-\vec{j}$$

$$\vec{a}+\vec{b}=(-2\vec{i}+3\vec{j})+(\vec{i}-\vec{j})=-2\vec{i}+3\vec{j}+\vec{i}-\vec{j}$$
$$=((-2)+1)\vec{i}+(3-1)\vec{j}=-1\vec{i}+2\vec{j}$$

$$\vec{a}-\vec{b}=(-2\vec{i}+3\vec{j})-(\vec{i}-\vec{j})=-2\vec{i}+3\vec{j}-\vec{i}+\vec{j}$$
$$=(-2-1)\vec{i}+(3+1)\vec{j}=-3\vec{i}+4\vec{j}$$

$$\cos\varphi=\frac{a_x b_x+a_y b_y}{\sqrt{{a_x}^2+{a_y}^2}\cdot\sqrt{{b_x}^2+{b_y}^2}}$$

$$\vec{a}+\vec{b}=-\vec{i}+2\vec{j}$$

$$\vec{a}-\vec{b}=-3\vec{i}+4\vec{j}$$

$$a_x=-1$$
$$a_y=2$$
$$b_x=-3$$
$$b_y=4$$

$$\cos\varphi=\frac{-1\cdot(-3)+2\cdot 4}{\sqrt{(-1)^2+2^2}\cdot\sqrt{(-3)^2+4^2}}=\frac{3+8}{\sqrt{1+4}\cdot\sqrt{9+16}}=\frac{11}{\sqrt{5}\cdot\sqrt{25}}$$
$$=\frac{11}{5\sqrt{5}}=0.98386991$$

ISPITNI ZADACI

$$\cos\varphi = 0.98386991$$

$$\varphi = 10^\circ\, 18'\, 17.45''$$

13. Izračunaj duljinu vektora $\vec{c} = \vec{a} - 3\,\vec{b}$ ako je $|\vec{a}| = \sqrt{8}$ $|\vec{b}| = 3$ i $\angle(\vec{a},\vec{b}) = 45^\circ$.

$\vec{c} = \vec{a} - 3\,\vec{b}$

$|\vec{a}| = \sqrt{8}$

$|\vec{b}| = 3$

$|\vec{a}| = \sqrt{8}\ /^2$

$\vec{a}^2 = \sqrt{8}^2 = 8$

$|\vec{b}| = 3/^2$

$\vec{b}^2 = 9$

$\vec{c} = \vec{a} - 3\,\vec{b}$

$|\vec{c}| = |\vec{a} - 3\,\vec{b}|^2$

$|\vec{a} - 3\,\vec{b}|^2 = \vec{a}^2 - 2\,\vec{a}\,3\,\vec{b} + (3\,\vec{b})^2 = \vec{a}^2 - 6 \cdot \vec{a} \cdot \vec{b} + (3\,\vec{b})^2 =$

$|\vec{a} - 3\,\vec{b}|^2 = 8 - 6 \cdot \vec{a} \cdot \vec{b} + 9 \cdot 9 = 8 - 6 \cdot \vec{a} \cdot \vec{b} + 81 =$

Skalarni umnožak $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$

$|\vec{a} - 3\,\vec{b}|^2 = 8 - 6 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \cos 45^\circ + 81 =$

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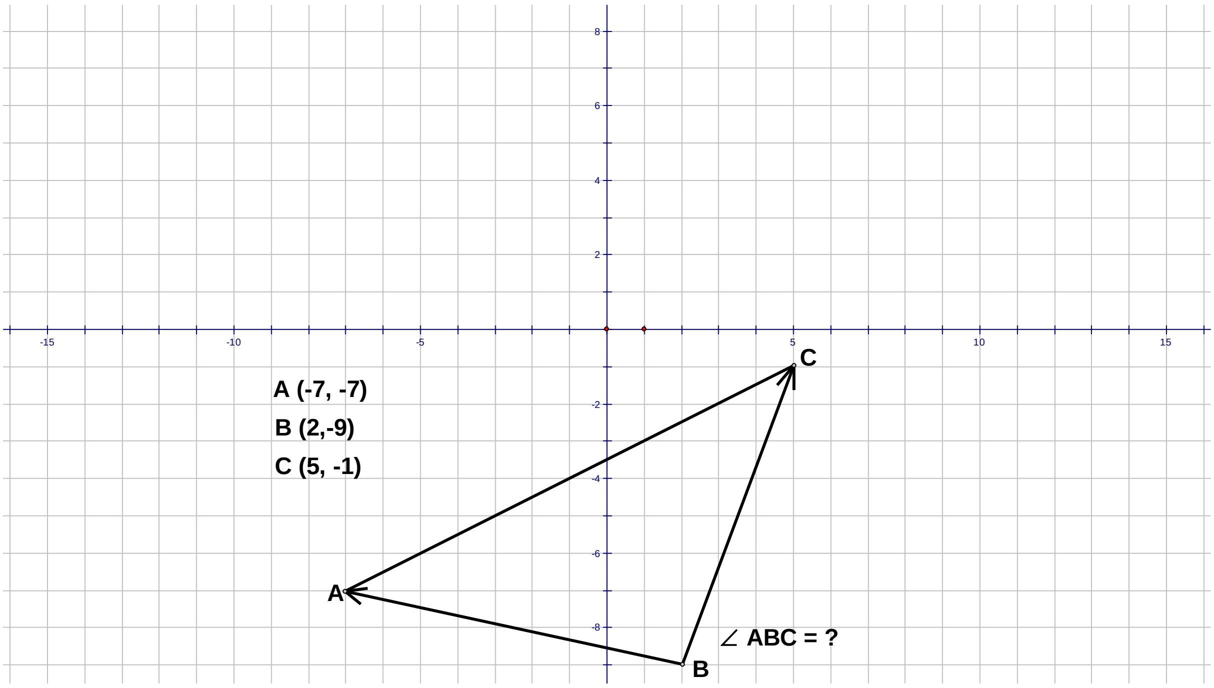
$$|\vec{a} - 3\,\vec{b}|^2 = 8 - 6 \cdot \sqrt{8} \cdot 3 \cdot \frac{\sqrt{2}}{2} + 81 =$$

$$|\vec{a} - 3\,\vec{b}|^2 = 8 - 3 \cdot \sqrt{16} \cdot 3 + 81 = 8 - 9 \cdot 4 + 81 = 8 - 36 + 81 = 53$$

14. Odredi najveći kut trokuta ABC ako je A (-7, -7), B (2, -9) i C (5, -1).



$$\overrightarrow{BA} = (x_A - x_B)\,\vec{i} + (y_A - y_B)\,\vec{j}$$

$$\overrightarrow{BA} = (-7 - 2)\,\vec{i} + \big(-7 - (-9)\big)\,\vec{j}$$

$$\overrightarrow{BA} = (-9)\,\vec{i} + 2\,\vec{j}$$

$$|\overrightarrow{BA}| = \sqrt{(BA)_x{}^2 + (BA)_y{}^2}$$

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$$|\overrightarrow{BA}| = \sqrt{(BA)_x{}^2 + (BA)_y{}^2} = \sqrt{(-9)^2 + 2^2} = \sqrt{81 + 4} = \sqrt{85}$$
$$= 9.219544457$$

$$\overrightarrow{BC} = (x_C - x_B)\,\vec{\imath} + (y_C - y_B)\,\vec{\jmath}$$

$$\overrightarrow{BC} = (5 - 2)\,\vec{\imath} + \big(-1 - (-9)\big)\,\vec{\jmath}$$

$$\overrightarrow{BC} = 3\,\vec{\imath} + (-10)\,\vec{\jmath} = 3\,\vec{\imath} - 10\,\vec{\jmath}$$

$$|\overrightarrow{BC}| = \sqrt{3^2 + (-10)^2} = \sqrt{9 + 100} = \sqrt{109} = 10.44030651$$

$$\cos\varphi = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|} = \frac{(-9)\cdot 3 + (2)\cdot(-10)}{\sqrt{85}\cdot\sqrt{109}} = \frac{-27 + (-20)}{\sqrt{85}\cdot\sqrt{109}}$$
$$= \frac{-47}{\sqrt{85}\cdot\sqrt{109}} = \frac{-47}{\sqrt{9265}} = -0.4882869822$$

$$\varphi = 119^\circ\, 13'\, 40.99''$$

15. Za koje su vrednoti realnog parametra m vektori $\vec{p} = 3\,\vec{\imath} + 4\,\vec{\jmath}$ $\vec{q} = m\,\vec{\imath} + 2\,\vec{\jmath}$ okomiti.

$$\vec{p} = 3\,\vec{\imath} + 4\,\vec{\jmath}$$
$$\vec{q} = m\,\vec{\imath} + 2\,\vec{\jmath}$$

$$p_x = 3$$
$$p_y = 4$$
$$q_x = m$$
$$q_y = 2$$

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Uvjet okomitosti:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = 0$$

$$\vec{p} \cdot \vec{q} = p_x q_x + p_y q_y = 0$$

$$\vec{p} \cdot \vec{q} = 3m + 4 \cdot 2 = 3m + 8 = 0$$

$$3m = -8$$
$$m = -\frac{8}{3}$$

16. Odredi vektor $\vec{b}$ okomit na vektor $\vec{a} = -2\vec{i} + \vec{j}$ i duljine $4\sqrt{5}$. Početak vektora $\vec{a}$ i vektora $\vec{b}$ su u ishodištu koordinatnog sustava.

$$|\vec{b}| = \sqrt{b_x^{\,2} + b_y^{\,2}} = 4\sqrt{5}$$

$$b_x^{\,2} + b_y^{\,2} = 4\sqrt{5} \,/^2$$

$$b_x^{\,2} + b_y^{\,2} = 4^2\sqrt{5}^2$$

$$b_x^{\,2} + b_y^{\,2} = 16 \cdot 5 = 80$$

$$b_x^{\,2} = 80 - b_y^{\,2}$$
$$b_x = \sqrt{80 - b_y^{\,2}}$$

Uvjet okomitosti:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = 0$$

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$$-2 \cdot \sqrt{80 - {b_y}^2} + 1 \cdot b_y = 0$$

$$2 \cdot \sqrt{80 - {b_y}^2} = b_y \ /^2$$

$$4 \cdot (80 - {b_y}^2) = {b_y}^2$$

$$4 \cdot 80 - 4 \cdot {b_y}^2 = {b_y}^2$$

$$320 - 4 \cdot {b_y}^2 - {b_y}^2 = 0$$

$$320 - 5\ {b_y}^2 = 0$$

$$320 = 5\ {b_y}^2$$

$${b_y}^2 = \frac{320}{5} = 64 / \pm\sqrt{\ }$$

$$b_y = \pm\sqrt{64}$$

$$\boldsymbol{b_y = \pm 8}$$

$$\boldsymbol{b_x} = \sqrt{80 - {b_y}^2} = \sqrt{80 - 8^2} = \sqrt{80 - 64} = \pm\sqrt{16} = \pm \mathbf{4}$$

$$\boldsymbol{b_x = \pm 4}$$

$$\vec{\boldsymbol{b}} = \mathbf{4}\,\vec{\boldsymbol{i}} + \mathbf{8}\,\vec{\boldsymbol{j}}$$

$$\vec{\boldsymbol{b}} = -\mathbf{4}\,\vec{\boldsymbol{i}} - \mathbf{8}\,\vec{\boldsymbol{j}}$$

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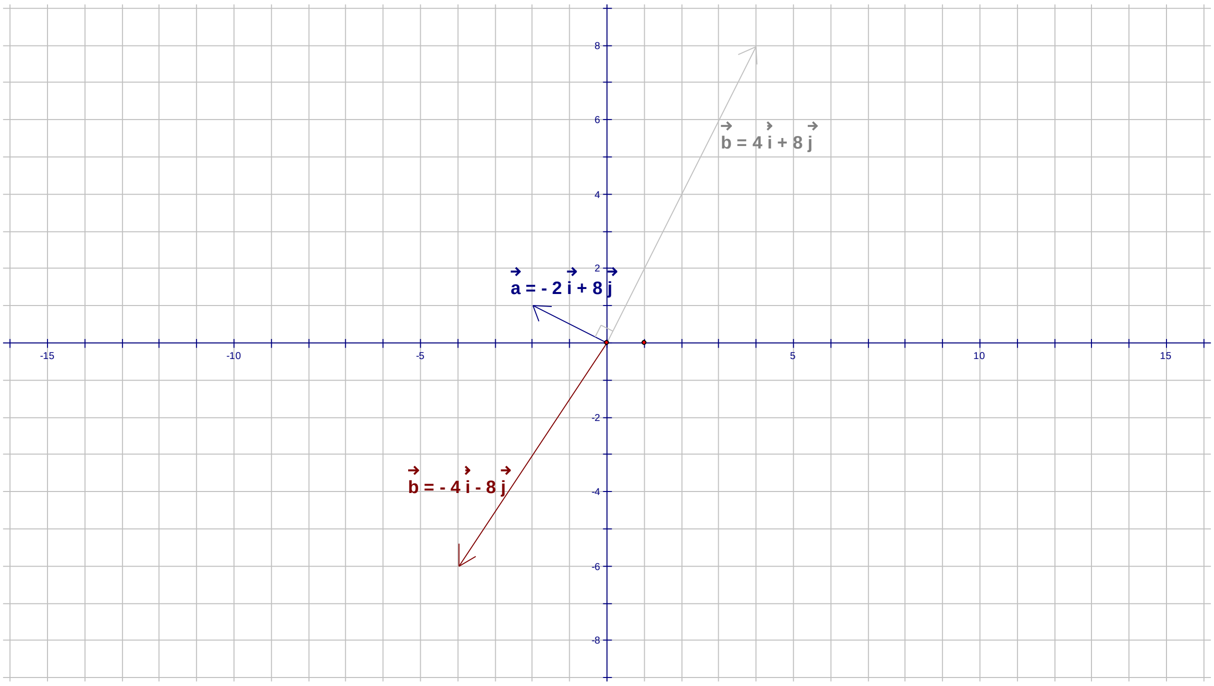

17. Težište T trokuta ΔABC leži na osi ordinata. Dva su vrha točke B (1, -2) i C (2, 5), a treći je vrh na osi apscisa. Odredi koordinate točaka A i T.

A $(x_A, 0)$

T (0, y_T)

$$x_T = \frac{x_A + x_B + x_C}{3}$$

$$y_T = \frac{y_A + y_B + y_C}{3}$$

$$x_T = \frac{x_A + 1 + 2}{3} = \frac{x_A + 3}{3} = 0$$

$$x_A + 3 = 0$$

$x_A = -3$ A (-3, 0)

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$$y_T = \frac{0+(-2)+5}{3} = \frac{-2+5}{3} = \frac{3}{3} = 1$$

$\boldsymbol{y_T} = 1$ T (0, 1)

18. Zadane su točke A (-3, 5), B (6, 7), C (1, -5). Odredi jedinične vektore u smjeru vektora $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{BC}$. Nađite koordinatu x točke T (x, 0) tako da vektori $\overrightarrow{TA}$ i $\overrightarrow{TB}$ budu okomiti.

Upute za rad:

Općenito jedinični vektor u smjeru vektora označavamo sa
$\vec{e} = m\,\vec{\imath} + n\,\vec{\jmath}$
Da bi vektor bio jedinični mora zadovoljavati:
$m^2 + n^2 = 1$

$$\overrightarrow{AB} = (x_B - x_A)\,\vec{\imath} + (y_B - y_A)\,\vec{\jmath}$$

$$\overrightarrow{AB} = (6-(-3))\,\vec{\imath} + (7-5)\,\vec{\jmath}$$

$$\overrightarrow{AB} = (6+3)\,\vec{\imath} + 2\,\vec{\jmath} = 9\,\vec{\imath} + 2\,\vec{\jmath}$$

Izračunati jedinični vektor u smjeru vektora $\overrightarrow{AB}$ po gore napisanim formulama:
$\vec{e}_{AB} = ?$

$$\overrightarrow{AC} = (x_C - x_A)\,\vec{\imath} + (y_C - y_A)\,\vec{\jmath}$$

$$\overrightarrow{AC} = (1-(-3))\,\vec{\imath} + (-5-5)\,\vec{\jmath}$$

$$\overrightarrow{AC} = (1+3)\,\vec{\imath} + (-5-5)\,\vec{\jmath}$$

$$\overrightarrow{AC} = 4\,\vec{\imath} + (-10)\,\vec{\jmath}$$

ISPITNI ZADACI

$\overrightarrow{AC} = 4\,\vec{\imath} - 10\,\vec{\jmath}$

$\vec{e}_{AC} = ?$

Izračunati jedinični vektor u smjeru vektora $\overrightarrow{AC}$ po gore napisanim formulama:

$\overrightarrow{BC} = (x_C - x_B)\,\vec{\imath} + (y_C - y_B)\,\vec{\jmath}$

$\overrightarrow{BC} = (1 - 6)\,\vec{\imath} + (-5 - 7)\,\vec{\jmath} = -5\,\vec{\imath} - 12\,\vec{\jmath}$

Izračunati jedinični vektor u smjeru vektora $\overrightarrow{BC}$ po gore napisanim formulama:

$\vec{e}_{BC} = ?$

Točki T (x, 0) treba naći koordinatu x

Uvjet okomitosti:

$$\overrightarrow{TA} \cdot \overrightarrow{TB} = (TA)_x\,(TB)_x + (TA)_y(TB)_y = 0$$

$$\overrightarrow{TA} = (x_A - x_T)\,\vec{\imath} + (y_A - y_T)\,\vec{\jmath} = (-3 - x_T)\,\vec{\imath} + (5 - 0)\,\vec{\jmath} = (-3 - x_T)\,\vec{\imath} + 5\,\vec{\jmath}$$

$$\overrightarrow{TB} = (x_B - x_T)\,\vec{\imath} + (y_B - y_T)\,\vec{\jmath} = (6 - x_T)\,\vec{\imath} + (7 - 0)\,\vec{\jmath} = (6 - x_T)\,\vec{\imath} + 7\,\vec{\jmath}$$

$$\overrightarrow{TA} \cdot \overrightarrow{TB} = (-3 - x_T) \cdot (6 - x_T) + 5 \cdot 7 = 0$$

$$\overrightarrow{TA} \cdot \overrightarrow{TB} = -18 + 3\,x_T - 6\,x_T + x_T^2 + 35 = 0$$

ISPITNI ZADACI

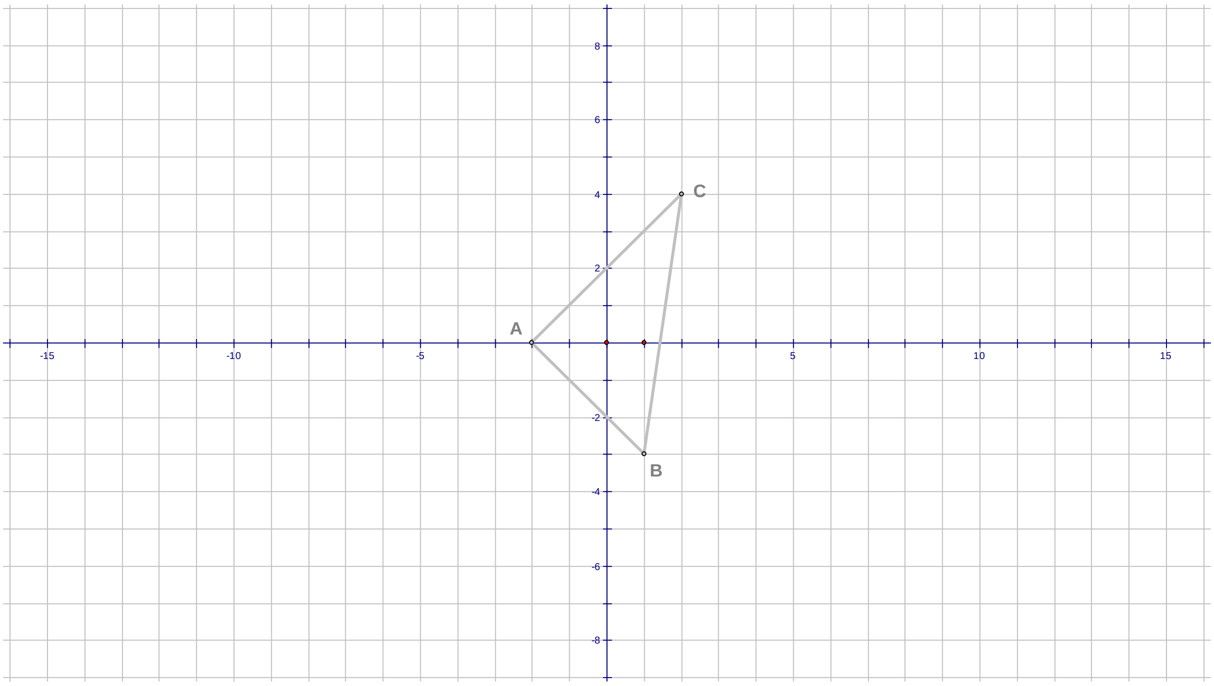
$$\overrightarrow{TA} \cdot \overrightarrow{TB} = 17 - 3\,x_T + x_T^2 = 0$$

$$x_T^2 - 3x_T + 17 = 0$$

Riješite kvadratnu jednadžbu: $x_{T1,2} = ?$

$T\,(x_{T1}, 0) = ?$
$T\,(x_{T2}, 0) = ?$

19. Ako su A (-2, 0), B (1, -3) i C (2, 4) vrhovi trokuta ΔABC, izračunajte opseg trokuta.



$$O = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{AC}|$$

$$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(1-(-2))^2 + ((-3) - 0)^2}$$
$$= \sqrt{(1+2)^2 + ((-3))^2} = \sqrt{(3)^2 + ((-3))^2} = \sqrt{9+9} = \sqrt{18}$$
$$= 3\sqrt{2}$$

ISPITNI ZADACI

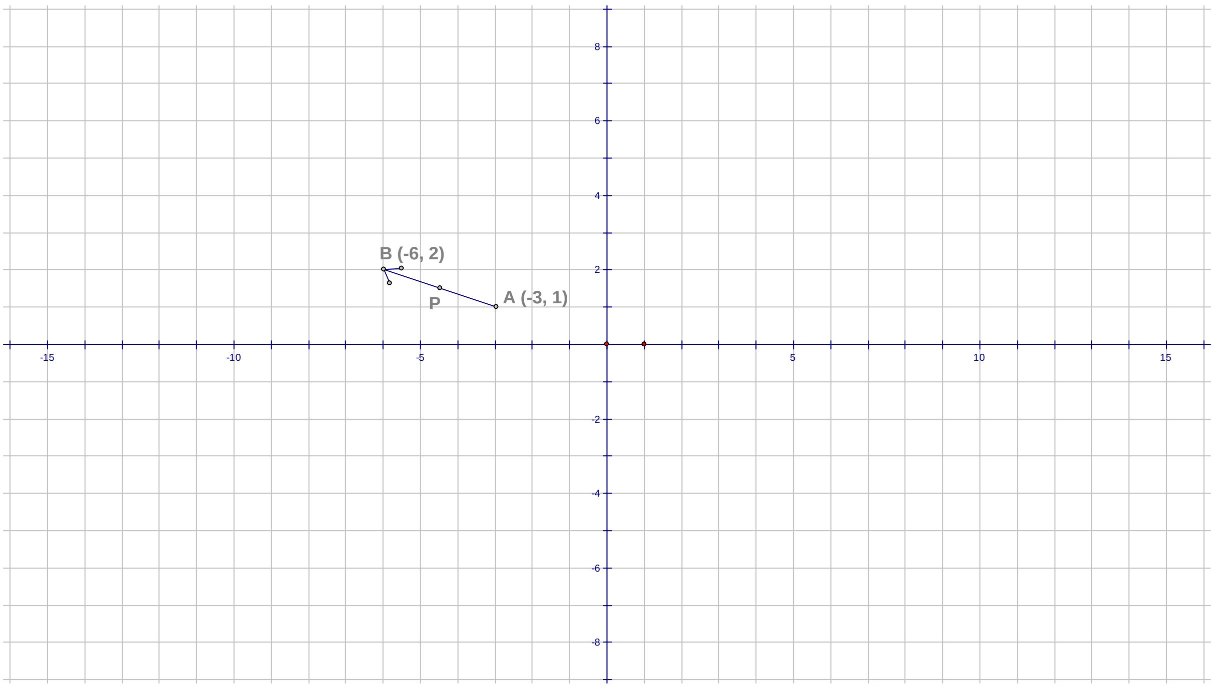
$$|\overrightarrow{BC}| = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(2-1)^2 + \left(4 - (-3)\right)^2}$$
$$= \sqrt{(1)^2 + (4+3)^2} = \sqrt{1^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$|\overrightarrow{AC}| = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{\left(2-(-2)\right)^2 + (4-0)^2}$$
$$= \sqrt{(2+2)^2 + 4^2} = \sqrt{(2+2)^2 + 4^2} = \sqrt{4^2 + 4^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$O = |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{AC}|$$

$$\mathbf{O} = 3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$$

20. Početak vektora $\overrightarrow{AB} = -6\vec{i} + 2\vec{j}$ je u točki (-3, 1), odredi polovište vektora $\overrightarrow{AB}$.



Polovište vektora $\overrightarrow{AB}$

A (-3, 1)

$A\ (x_A, y_A)$

B (-6, 2)

ISPITNI ZADACI

$B\ (x_B\ , y_B)$

$$\boldsymbol{x_P = \frac{x_A + x_B}{2}}$$

$$x_P = \frac{-3 + (-6)}{2} = \frac{-3 - 6}{2} = \frac{-9}{2} = -4.5$$

$$\boldsymbol{y_P = \frac{y_A + y_B}{2}}$$

$$y_P = \frac{1 + 2}{2} = \frac{3}{2} = 1.5$$